2.4: Reduced Row Echelon Form

Reduced Row Echelon Form

A matrix $A \in M_{m,n}(\mathbb{R})$ is in **reduced row echelon form**, or **rref**, if all of the following are true of A:

1. every leading entry of A is a leading 1,
2. all rows of A consisting of only zeros occur at the bottom of the matrix,
3. every leading 1 of A occurs farther to the right than the leading 1 in the preceding row,
4. every entry of A above or below a leading 1 is equal to 0.

2.4: Converting Matrices to RREF

- Every matrix is row equivalent to a unique RREF.
- The process for converting a matrix to RREF is called **Gauss-Jordan Elimination**.
- This process is given in general in the book. In class, it will be much easier to describe the process using a series of examples.

2.4: Solving Linear Systems Using RREF

Procedure for Solving a Linear System in rref:

Let rref $\left(A|\vec{\mathbf{b}}\right)$ be the rref of the augmented matrix of a linear system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

- If rref $\left(A|\vec{\mathbf{b}}\right)$ has a leading 1 in the last column (the column to the right of the bar), then the linear system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is *inconsistent* and has *no solution*.
- If rref $\left(A|\vec{\mathbf{b}}\right)$ has all of its leading 1s to the left of the bar and every column of rref $\left(A|\vec{\mathbf{b}}\right)$ left of the bar has a leading 1, then the linear system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has a *unique solution*. This unique solution may be determined by converting rref $\left(A|\vec{\mathbf{b}}\right)$ back into a linear system in the original variables.

2.4: Solving Linear Systems Using RREF

Procedure for Solving a Linear System in rref (cont.):

- Suppose that all of the leading 1s in rref $\left(A|\vec{\mathbf{b}}\right)$ are to the left of the bar, and that there are columns to the left of the bar with no leading 1s. The columns to the left of the bar with no leading 1s are called **free columns** or **non-leading columns**. In this case, the linear system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has *infinitely many solutions*:
  1. Each of the original variables corresponding to free columns in rref $\left(A|\vec{\mathbf{b}}\right)$ is a **free variable**, and can be chosen to have any real value. Such free variables are usually assigned parameters such as $s, t, \ldots$
  2. The original variables corresponding to non-free columns in rref $\left(A|\vec{\mathbf{b}}\right)$ are called **non-free variables**. Every equation in the system can be solved for a non-free variable in terms of the parameters.